The results of the calculation of the functions $\Omega_n(r^*)$ in the first approximation for Pr = 1, 7.03, 50, and 100 are shown in Fig. 2. The weak dependence of the functions $\Omega_n(r^*)$ on the Prandtl number is seen from the figure.

NOTATION

θ, dimensionless temperature; $a_s(a_\infty)$, coefficient of thermal diffusivity of plate (of onflowing gas); $\lambda_s(\lambda_\infty)$, coefficient of thermal conductivity of plate (of onflowing gas); L(H), length (thickness) of plate; C, constant in Chapman—Rubezin law.

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CONJUGATE PROBLEM OF STEADY HEAT EXCHANGE IN THE LAMINAR FLOW OF AN INCOMPRESSIBLE FLUID IN A FLAT CHANNEL

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UDC 536.244

The conjugate problem of convective heat exchange in a flat channel is solved by the combined application of the integral Laplace transformation and the Bubnov—Galerkin method.

Considerable attention has recently been paid to problems of conjugate heat exchange [1].

Results of numerical calculations of the conjugate problem of steady heat exchange are presented in [2, 3], while an exact solution of the very general problem of conjugate heat exchange with boundary conditions of the first kind at the surface of the pipe wall is obtained analytically in [4]. However, practical use of the solutions obtained is made difficult by the complicated and cumbersome functional dependences.

The combined application of the integral Laplace transformation and the Bubnov—Galerkin method [5] allows one to obtain an approximate solution of the conjugate problem which is suitable for direct calculations.

Let us make the following assumptions: the flow of the fluid and the process of heat exchange are steady; the heat-transfer agent is viscous and incompressible; the mode of flow is laminar; the temperature of the heat-transfer agent is constant in the entrance section of the channel; the temperature of the outer surface of the channel walls is an arbitrary function of the longitudinal coordinate; the curvature of the temperature distribution in the fluid in the longitudinal direction can be neglected in comparison with the curvature in the transverse direction — this assumption is evaluated in [6]; the temperature field is axisymmetric. With allowance for these assumptions the energy equation for the fluid in dimensionless variables has the form

$$\frac{3}{2}(1-\xi^2)\frac{\partial\Theta_1(\xi, X)}{\partial X} = \frac{\partial^2\Theta_1(\xi, X)}{\partial\xi^2} \quad (0<\xi<1,\ 0<X<\infty). \tag{1}$$

We assume that the channel wall is made of an anisotropic material (here the coordinate system coincides with the principal coordinate system), and then the heat-conduction equation for the wall is written in the form

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TABLE 1. Roots p_j and Coefficients β_j and γ_{jk}

j	p_j	β_j	γ_{j1}	γ_{j2}	γ_{j3}
			$K_s=0,1\cdot10^4$		
1	—1,88279	—0,001713	—1,194579	+0,486489	—0,148417
2	—22,3044	—0,000765	+0,218081	—3,408139	+2,807211
3	—150,512	—0,008401	—0,043503	+2,921745	—7,078958
			$K_s=0,1\cdot10^3$		
1	—1,854163	—0,016922	—1,177969	+0,475693	—0,143859
2	—22,15240	—0,007546	+0,224573	—3,372003	+2,770097
3	—140,16118	—0,074629	+0,024345	+2,896314	—6,652123
			$K_s=0,5\cdot10^2$		
1	—1,82326	—0,03391	—1,15989	+0,4642	—0,13901
2	—21,98723	—0,01484	+0,23143	—3,33096	+2,72848
3	—130,65960	—0,13209	+0,08317	+2,86684	—6,25236
			$K_s=0,1\cdot10^2$		
1	—1,60627	—0,15024	—1,02971	+0,38569	—0,10719
2	—20,80665	—0,06372	+0,27349	—2,99039	+2,39835
3	—91,04984	—0,30986	+0,26515	+2,60470	—4,41913
			$K_s=0,1\cdot10^1$		
1	—0,667084	—0,660217	—0,4280376	+0,1153458	—0,0194166
2	—15,99977	—0,121429	+0,230754	—1,073961	+0,782492
3	—49,89362	—0,135020	+0,1113456	+0,958615	—1,1374709

$$\frac{\partial^2\Theta_2(\xi, X)}{\partial\xi^2} + N\frac{1}{\mathrm{Pe}^2}\cdot\frac{\partial^2\Theta_2(\xi, X)}{\partial X^2} = 0,$$

where $N = (\lambda_{xx}/\lambda_{yy})$ is the ratio of thermal conductivities.

If the condition $X \gg (\sqrt{N}/\mathrm{Pe})\xi$ is satisfied, then the second term in the latter equation can be neglected and the heat-conduction equation is simplified:

$$\frac{\partial^2\Theta_2(\xi, X)}{\partial\xi^2} = 0 \quad (1<\xi<\delta). \tag{2}$$

Such cases are often encountered in engineering practice, for example, in calculating the temperature fields of the layered windings of transformers [7].

For a complete formulation of the problem we add the following boundary conditions to the differential equations (1) and (2):

$$\Theta_1(\xi, 0) = 0, \tag{3}$$

$$\frac{\partial\Theta_1(0, X)}{\partial\xi} = 0, \tag{4}$$

$$\Theta_1(1, X) = \Theta_2(1, X), \tag{5}$$

$$\left.\frac{\partial\Theta_1(\xi, X)}{\partial\xi}\right|_{\xi=1} = K_\lambda \left.\frac{\partial\Theta_2(\xi, X)}{\partial\xi}\right|_{\xi=1}, \tag{6}$$

$$\Theta_2(\delta, X) = f(X). \tag{7}$$

We apply the integral Laplace transformation with respect to the variable X to the boundary problem (1)-(7) and then in the transform region we obtain

$$\frac{3}{2}(1-\xi^2)\,p\bar{\Theta}_1(\xi, p) = \frac{d^2\bar{\Theta}_1(\xi, p)}{d\xi^2}, \tag{8}$$

$$\frac{d^2\bar{\Theta}_2(\xi, p)}{d\xi^2} = 0, \tag{9}$$

$$\frac{d\bar{\Theta}_1(0, p)}{d\xi} = 0, \tag{10}$$

$$\bar{\Theta}_1(1, p) = \bar{\Theta}_2(1, p) = \bar{\chi}(p), \tag{11}$$

TABLE 2. Coefficients Q_j and W_j

	$K_s=0{,}1\cdot10^1$		$K_s=0{,}1\cdot10^2$	
j	Q_j	W_j	Q_j	W_j
1	—1,320433	—0,990128	—3,00485	—0,93401
2	—0,242859	—0,029755	—1,27422	—0,095314
3	—0,270040	+0,020281	—6,19709	+0,04160

	$K_s=0{,}1\cdot10^3$		$K_s=0{,}1\cdot10^4$	
j	Q_j	W_j	Q_j	W_j
1	—3,23845	—0,9104138	—3,42603	—0,907432
2	—1,50933	—0,1077325	—1,53139	—0,108861
3	—14,9258	+0,896065	—16,8029	—0,810712

$$\frac{d\bar{\Theta}_1(1,\ p)}{d\xi} = K_\lambda \frac{d\bar{\Theta}_2(1,\ p)}{d\xi}, \tag{12}$$

$$\bar{\Theta}_2(\delta,\ p) = \bar{f}(p), \tag{13}$$

where $\bar{\chi}(p)$ is an unknown function of the temperature of the system when $\xi = 1$ in the transform region.

The solution of the problem (9), (11), (13) presents no difficulties and has the form

$$\bar{\Theta}_2(\xi,\ p) = \bar{\chi}(p) + \frac{\bar{f}(p) - \bar{\chi}(p)}{\delta - 1}(\xi - 1). \tag{14}$$

Now we can determine the temperature of the fluid using the Bubnov — Galerkin method. We represent the solution of Eq. (8) in the form

$$\bar{\Theta}_1(\xi,\ p) = \bar{\chi}(p) + \sum_{k=1}^{N} \bar{a}_k(p)\,\omega_k(\xi)$$

and for the approximation of the liquid temperature distribution with respect to ξ we use the complete system of linearly independent functions

$$\omega_k(\xi) = (1 - \xi^2)\,\xi^{2(k-1)}.$$

This choice of the coordinate functions makes it possible to represent the solution in a form convenient for calculations and to satisfy the boundary conditions of the problem.

Following integration, the condition of orthogonality of the discrepancy to the coordinate functions leads to a system of N algebraic equations in which the unknowns are $\bar{a}_k(p)$:

$$\sum_{j=1}^{N} \bar{a}_j(p)(A_{kj} + pC_{kj}) = p\bar{\chi}(p)\,E_k, \tag{15}$$

where the rational numbers A_{kj}, C_{kj}, and E_k are determined from the equations

$$A_{kj} = \int_0^1 \omega_j''(\xi)\,\omega_k(\xi)\,d\xi,$$

$$C_{kj} = -\frac{3}{2}\int_0^1 (1 - \xi^2)\,\omega_k(\xi)\,\omega_j(\xi)\,d\xi,$$

$$E_k = \int_0^1 \frac{3}{2}(1 - \xi^2)\,\omega_k(\xi)\,d\xi.$$

Having used the Cramer rule to determine the coefficients $\bar{a}_k(p)$, we obtain the temperature distribution of the liquid in the transform region:

$$\bar{\Theta}_1(\xi,\ p) = \bar{\chi}(p) - \sum_{k=1}^{N} p\bar{\chi}(p)\frac{\Delta_k(p)}{\Delta(p)}\,\omega_k(\xi), \tag{16}$$

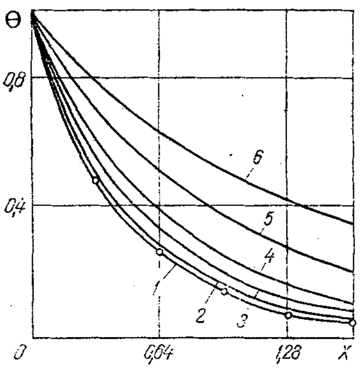

Fig. 1

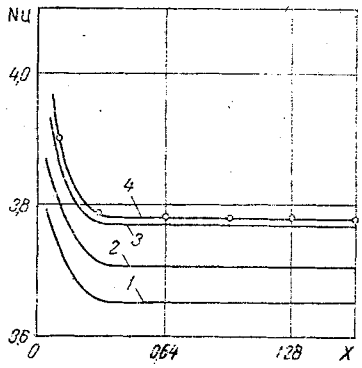

Fig. 2

Fig. 1. Variation in $\overline{\Theta}(X)$ along length of flat slot (solid curve: calculations by Eq. (22), points: results of [6]); 1) $K_S = \infty$; 2) 100; 3) 10; 4) 5; 5) 2; 6) 1.

Fig. 2. Dependence of Nu on X for a flat slot (solid curve: calculation by Eq. (24), points: results of [5]): 1) $K_S = 50$; 2) 10; 3) 1000; 4) ∞.

where $\Delta(p)$ and $\Delta_k(p)$ ($k = 1, 2, \ldots, N$) are the principal and auxiliary determinants, respectively, of the algebraic system (15).

Using the solutions (14) and (16), we determine the function $\bar{\chi}(p)$ from the condition (12) of equality of the heat fluxes at the conjugation surface:

$$\bar{\chi}(p) = \frac{K_s}{K_s\Delta(p) + 2p\sum\limits_{j=1}^{N}\Delta_j(p)}\,\Delta(p)\,f(p). \tag{17}$$

Here $K_S = K\lambda/(\delta - 1)$ is the similarity criterion, which we call the conjugation coefficient.

By means of an integral Laplace transformation, with allowance for (14), (16), and (17), in the transform region we obtain the temperature distribution in the form

$$\overline{\Theta}_1(\xi, p) = \bar{f}(p) - 2p\bar{f}(p)\sum_{j=1}^{N}\Delta_j(p)\,\frac{1}{\psi(p)} - K_s p\bar{f}(p)\,\frac{1}{\psi(p)}\sum_{j=1}^{N}\Delta_j(p)\,\omega_j(\xi), \tag{18}$$

$$\overline{\Theta}_2(\xi, p) = \bar{f}(p) - 2pf(p)\sum_{j=1}^{N}\Delta_j(p)\,\frac{1}{\psi(p)}\cdot\frac{\delta-\xi}{\delta-1}, \tag{19}$$

where

$$\psi(p) = K_s\Delta(p) + 2p\sum_{j=1}^{N}\Delta_j(p).$$

The justification of the possibility of an inverse integral Laplace transformation in Eqs. (18) and (19) raises no difficulties (see [5], for example). In doing this we use the Borel theorem [8] and expansion theorems [8], and then the solution of problem (1)-(7) is written in the form

$$\Theta_1(\xi, X) = f(X) + \sum_{j=1}^{N}\beta_j\int_0^X f^*(t)\exp[p_j(X-t)]\,dt + \sum_{j=1}^{N}(1-\xi^2)\int_0^1 f^*(t)\exp[p_j(X-t)]\,dt\sum_{k=1}^{N}\gamma_{jk}\xi^{2(k-1)}, \tag{20}$$

$$\Theta_2(\xi, X) = f(X) + \frac{\delta-\xi}{\delta-1}\sum_{j=1}^{N}\beta_j\int_0^X f^*(t)\exp[p_j(X-t)]\,dt, \tag{21}$$

where

$$\beta_j = \frac{-2\sum\limits_{k=1}^{N}\Delta_k(p_j)}{\psi'(p_j)},$$

$$\gamma_{jk} = \frac{-K_s\Delta_k(p_j)}{\psi'(p_j)};$$

p_j are the real roots of the equation $\psi(p) = 0$; $f^*(X)$ is the inverse of the function $p\bar{f}(p)$ by a Laplace transform.

To facilitate the calculations, the values of the coefficients β_j and γ_{jk} and the roots p_j for the case of N = 3 for different values of the conjugation coefficient K_s are presented in Table 1.

The solutions obtained were tested for a constant temperature $t_w \neq t_0$ of the outer surface of the wall.

Using the solutions (20) and (21) we calculated the following: the dimensionless average-integral temperature of the fluid

$$\bar{\Theta}(X) = \frac{\bar{t}(x) - t_w}{t_0 - t_w} = \frac{3}{2} \int_0^1 \Theta_1(X, \xi)(1 - \xi^2)\, d\xi, \tag{22}$$

the temperature of the inner surface of the wall

$$\Theta_1(X, 1) = \Theta_2(X, 1), \tag{23}$$

and the local Nusselt number

$$\mathrm{Nu}(X) = \frac{-2\left(\frac{\partial \Theta_1}{\partial \xi}\right)\Big|_{\xi=1}}{\bar{\Theta}(X)} = \frac{\sum_{j=1}^{N} Q_j \exp(p_j X)}{\sum_{k=1}^{N} W_k \exp(p_k X)}, \tag{24}$$

where to facilitate the calculations the coefficients Q_j and W_k are presented in Table 2 for different values of the conjugation coefficient K_s.

The dependences $\bar{\Theta}(X)$ and Nu(X) calculated from Eqs. (22) and (24) are presented in Figs. 1 and 2, respectively, for different values of the conjugation coefficient K_s.

In the limiting case for the value of the coefficient K_s, as $K_s \to \infty$ (the wall thickness approaches zero), the curves of $\bar{\Theta}(X)$ and Nu(X) give good agreement with the calculated results of [5, 6].

An analysis of the results of the calculations shows that variation in the thickness of the channel wall and in the ratios of thermal conductivities of the wall and the heat-transfer agent has a considerable effect on the average-mass temperature and the local Nusselt number. Thus, the limiting value Nu_∞ of the Nusselt number calculated without allowance for heat conduction or the wall thickness ($K_s = \infty$) exceeds the limiting Nusselt number Nu_∞ with $K_s = 0.5 \cdot 10^2$ by almost 4%.

This disagreement increases with a decrease in the criterion K_s.

As seen from the solutions obtained, with the assumptions which were made the condition of similarity of steady processes of heat exchange in a fluid flowing in a flat channel is the equality of the criteria

$$K_s = \frac{\lambda_2}{\lambda_1} \frac{R_1}{R_2 - R_1}.$$

Calculations based on the solutions (21) and (24) and the data of Figs. 1 and 2 allow one to select the optimum thickness of the wall and material for its fabrication.

We have also solved the problem of conjugate heat exchange with allowance for frictional heat by the method suggested.

NOTATION

ξ, X, transverse and longitudinal dimensionless coordinates; $\Theta_1(\xi, X)$, $\Theta_2(\xi, X)$, dimensionless temperatures of fluid and of solid wall; f(X), temperature at outer surface of wall; $2R_1$, width of channel; $2R_2$, distance between outer surfaces of channel walls; $K_\lambda = \lambda_2/\lambda_1$, ratio of thermal conductivities of wall material and heat-transfer agent; $\bar{\Theta}_1$, $\bar{\Theta}_2$, corresponding functions in the region of Laplace transforms; A_{kj}, C_{kj}, E_k, β_k, γ_{jk}, constants; $K_s = K_\lambda[R_1/(R_2 - R_1)]$, criterion of conjugate heat exchange.

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EFFECT OF TURBULENCE OF THE OUTER STREAM ON THE TRANSITION FOR SOME CLASSES OF SELF-SIMILAR FLOWS

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It is shown that in the presence of very low levels of pulsations at the outer limit of the laminar boundary layer the energy of neutral oscillations within the layer itself reaches very high values. This predetermines the transition to turbulent flow.

1. Primary Motion and Equation of the Oscillations

For the study of the stability of laminar flow, as is usually done in mechanics in the theory of stability, we will examine the primary motion on which the perturbed motion, caused by the presence of disturbances in the form of a degree of turbulence E_0 of the outer stream, is superimposed. In the general case one can assume that the perturbed motion affects the primary motion. The equations of the plane motion of a viscous incompressible fluid, written through the stream function $\psi(x, y, t)$, have the form [1]

$$\frac{\partial}{\partial t}(\Delta\psi)-\frac{\partial\psi}{\partial x}\frac{\partial}{\partial y}(\Delta\psi)+\frac{\partial\psi}{\partial y}\frac{\partial}{\partial x}(\Delta\psi)=\nu\Delta\Delta\psi. \tag{1}$$

The stream function $\psi(x, y, t)$ describes the instantaneous state of the liquid or gas. We designate the primary motion through $\Psi(x, y)$, while the stream function $\psi'(x, y, t)$ will describe the perturbed motion. Then $\psi = \Psi + \psi'$, where the stream function ψ' can be determined from the equation

$$(\Delta\psi')_t+\Psi_y(\Delta\psi')_x+\psi'_y(\Delta\Psi)_x-\Psi_x(\Delta\psi')_y-\psi'_x(\Delta\Psi)_y=\nu\Delta\Delta\psi'. \tag{2}$$

Considering the motion in the boundary layer, by substituting $\psi = \Psi + \psi'$ into Eq. (1) and carrying out the averaging (in the ergodic sense) of the equation obtained we arrive at the average equation of the primary motion in the form [2]

$$uu_x+vu_y+\overline{u'^2_x}+\overline{u'v'_y}=u_eu_{ex}+\nu u_{yy}, \tag{3}$$

where

$$u=\Psi_y;\quad v=-\Psi_x;\quad u'=\psi'_y;\quad v'=-\psi'_x.$$

The last two terms on the left side of (3) characterize the effect of the perturbed motion on the primary motion.

Henceforth, solutions for system (3) like those which follow flow of the Folkner – Skan type, i.e., $u_e = cx^m$, where the index e pertains to the outer nonviscous stream, will be examined as solutions of the equations

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